Functions

Consider the function f defined over D=IR by $f(x) = x^4 + 2x^3$.(C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1. Calculate the limits of f at the boundaries of D.

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x^4 = (+\infty)^4 = +\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^4 = (-\infty)^4 = +\infty$$

2. Solve $f(x) = 0$ and interpret graphically.

$f(x) = 0$; $x^4 + 2x^3 = 0$

$x^3(x+2) = 0$

$x = 0$ or $x + 2 = 0$

$x = -2$

Graphical interpretation:

(C) Cuts $(x'x)$ at 2 points: (0;0) and (-2;0).

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3. Find $f'(x)$ and set up the table of variations of f.

$f'(x) = 4x^3 + 6x^2$

$f'(x) = 0 \quad ; \quad 2x^2(2x+3) = 0$

$x = 0$ or $2x + 3 = 0$

$x = -\frac{3}{2}$

x	$-\infty$		$-\frac{3}{2}$		0		$+\infty$
$f'(x)$		$-$	0	$+$	0	$+$	
$f(x)$	$+\infty$	↘	-1.69	↗	0	↗	$+\infty$

$f(0) = 0^5 + 2(0)^3 = 0$

$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^4 + 2\left(-\frac{3}{2}\right)^3 = -1.69$

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4. Write the equation of the tangent (T) at point A of abscissa 0. and study the relative position between (T) and (C).

At $x = 0$: $f'(0) = 0$ so (T) is a horizontal line

Then the equation of the tangent (T) is $y = y_A = 0$

Hence, (T) is $(x'x)$.

Relative position:

$$f(x) - y_{(T)} = x^4 + 2x^3 - 0$$

$$= x^4 + 2x^3 = x^3(x+2)$$

$x = 0$ or $x = -2$

x		-2		0	
x^3	$-$		$-$	0	$+$
$x+2$	$-$	0	$+$		$+$
$f(x) - y$	$+$	0	$-$	0	$+$
Position	(C) above (T)	(C) cuts (T) At (-2;0)	(C) below (T)	(C) cuts (T) At (0;0)	(C) above (T)

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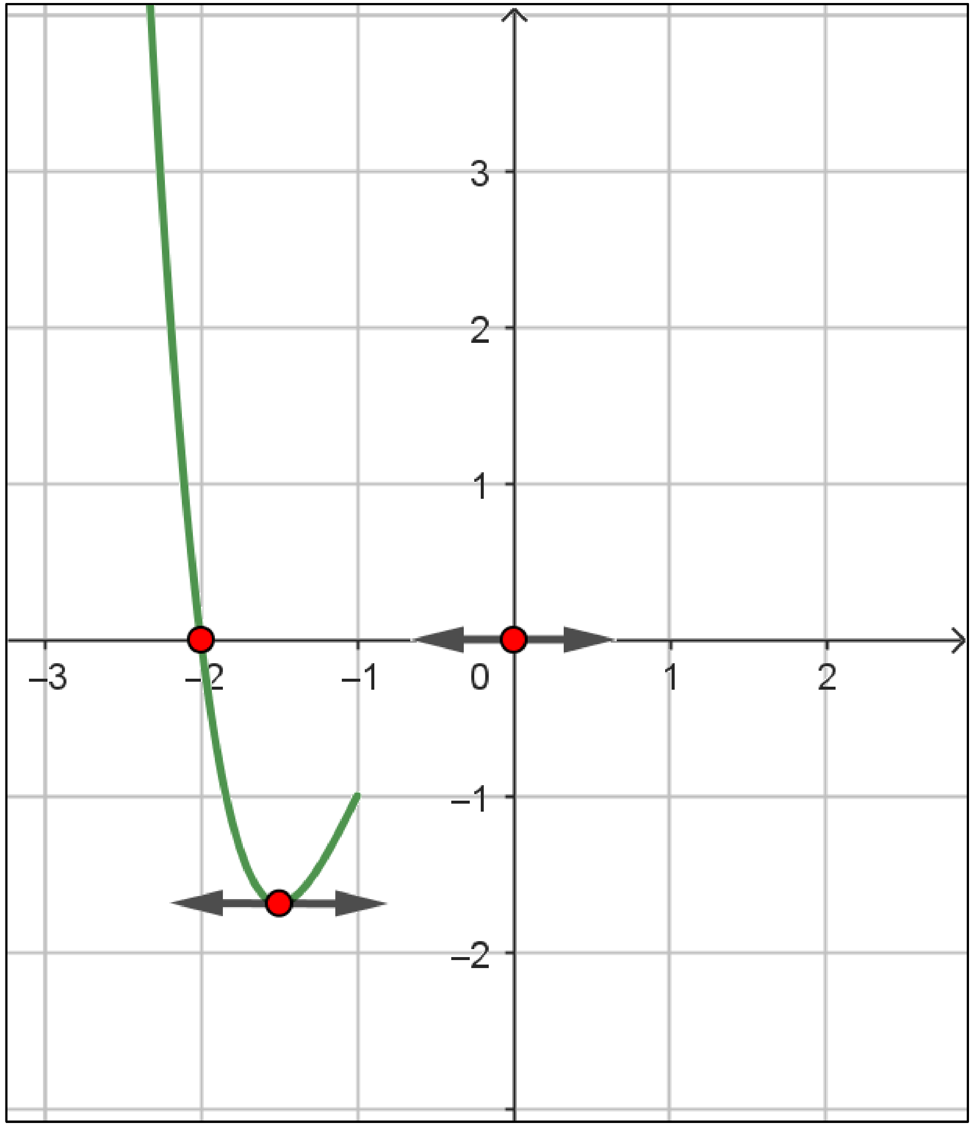
5. Plot (C).



x	$-\infty$		$-\frac{3}{2}$		0		$+\infty$
$f'(x)$		$-$	0	$+$	0	$+$	
$f(x)$	$+\infty$	↘	-1.69	↗	0	↗	$+\infty$

- Start by the extrema
- Plot Particular points
- Start drawing the curve based on the table of variations.

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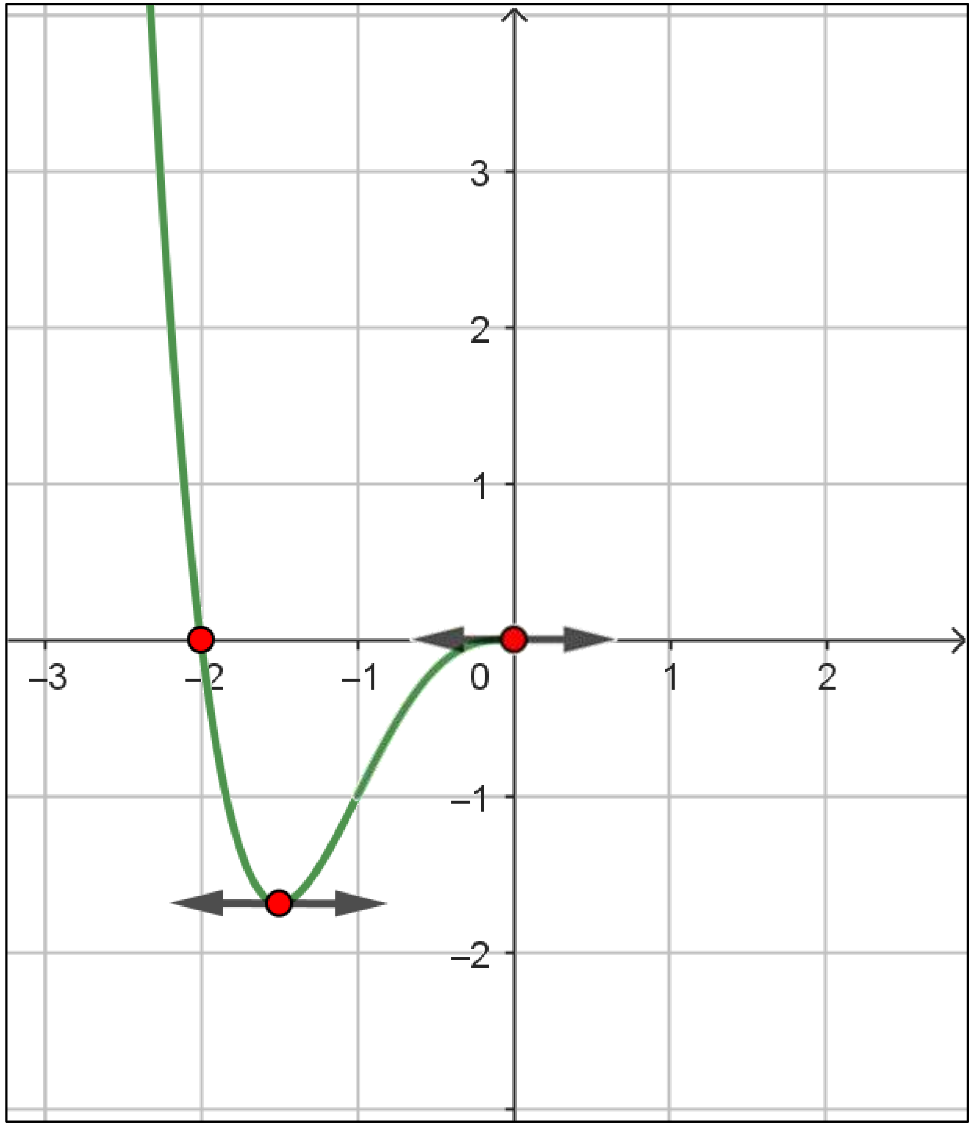

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$f'(x)$		$-$	0	$+$	0	$+$	
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x		-2		0	
x^3	$-$		$-$	0	$+$
$x+2$	$-$	0	$+$		$+$
$f(x) - y$	$+$	0	$-$	0	$+$
Position	(C) above (T)	(C) cuts (T) At (-2;0)	(C) below (T)	(C) cuts (T) At (0;0)	(C) above (T)

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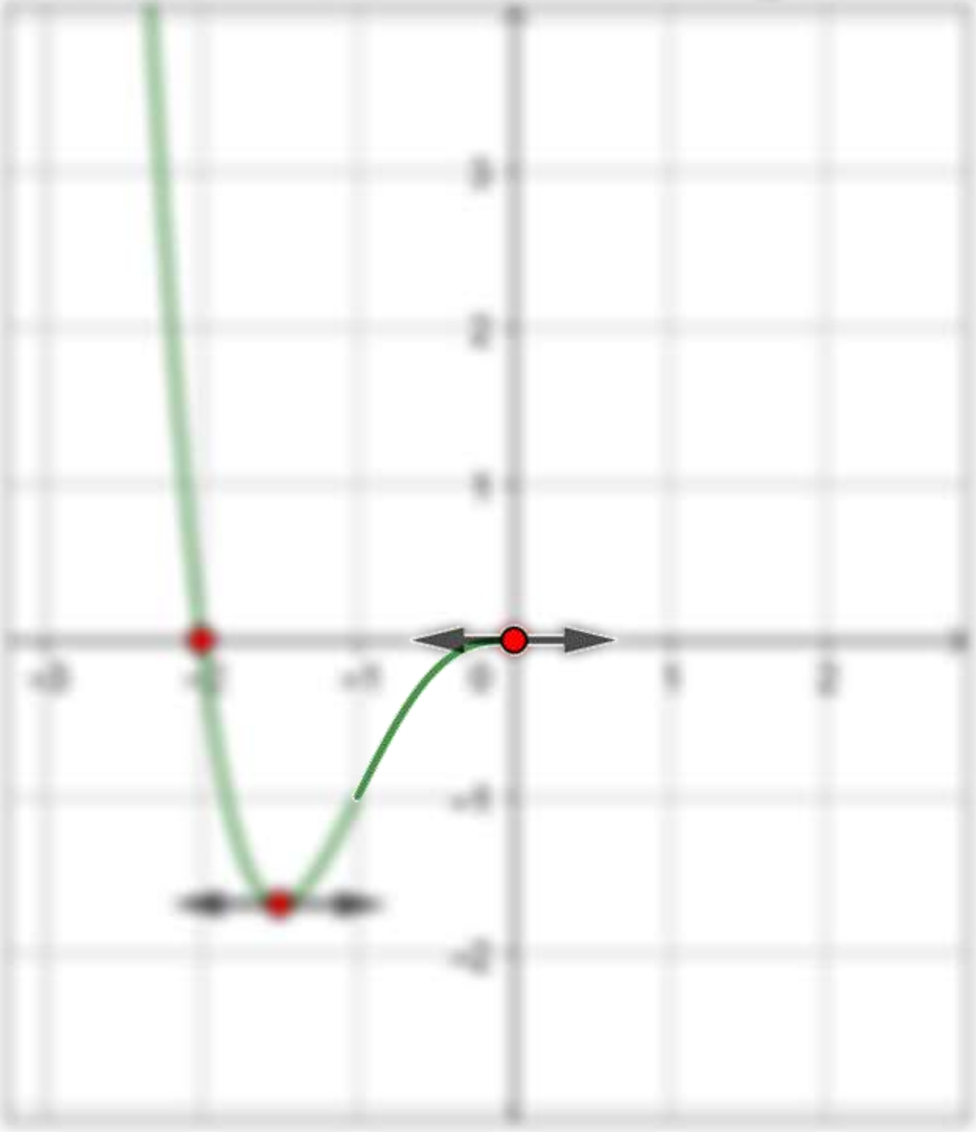
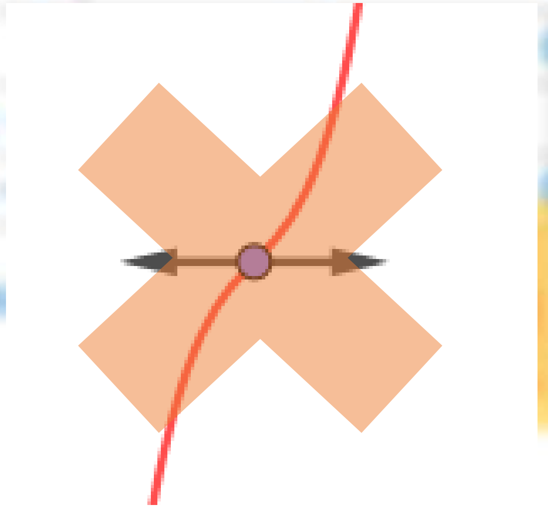


Pay attention the curve must be tangent to the horizontal tangent.



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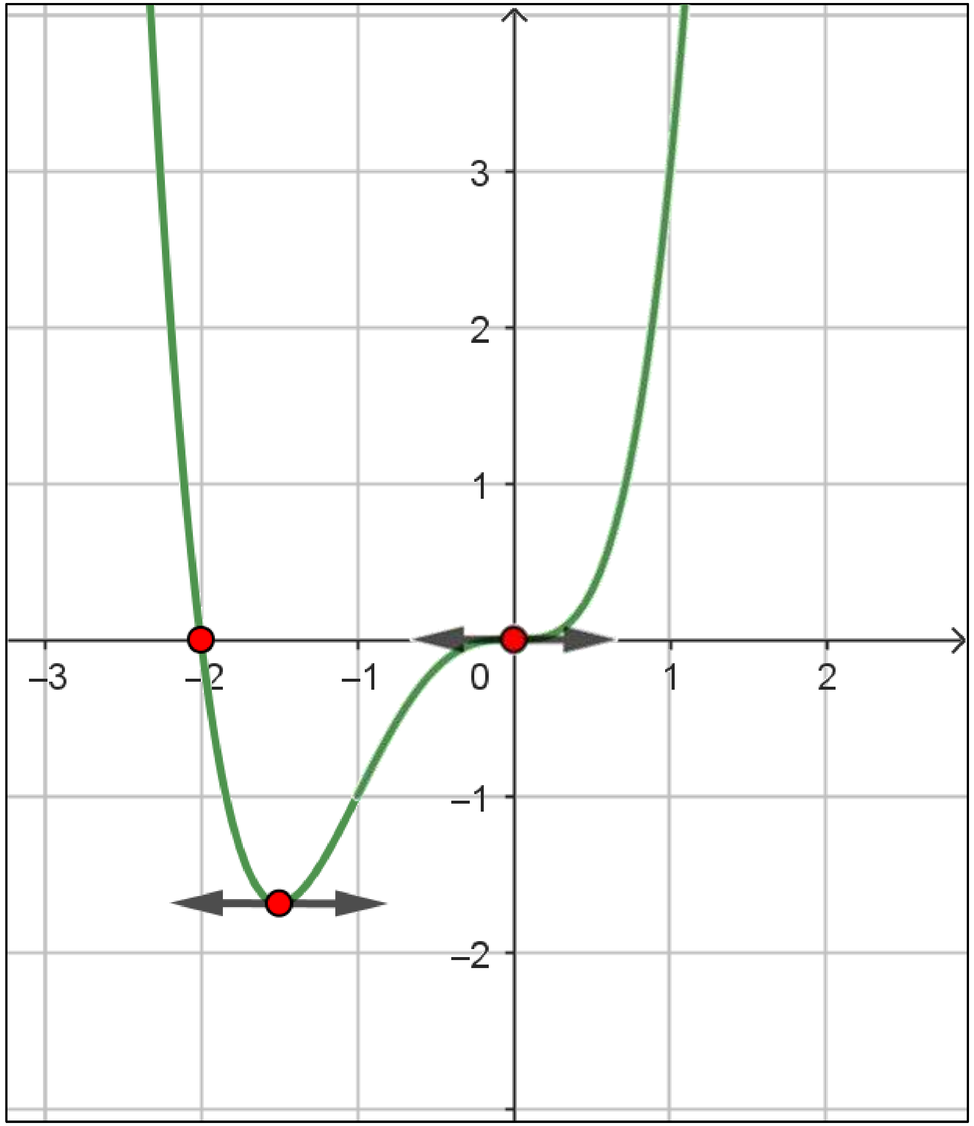

x	$-\infty$		$-\frac{3}{2}$		0		$+\infty$
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x		-2		0	
x^3	$-$		$-$	0	$+$
$x+2$	$-$	0	$+$		$+$
$f(x)-y$	$+$	0	$-$	0	$+$
Position	(C) above (T)	(C) cuts (T) At (-2;0)	(C) below (T)	(C) cuts (T) At (0;0)	(C) above (T)

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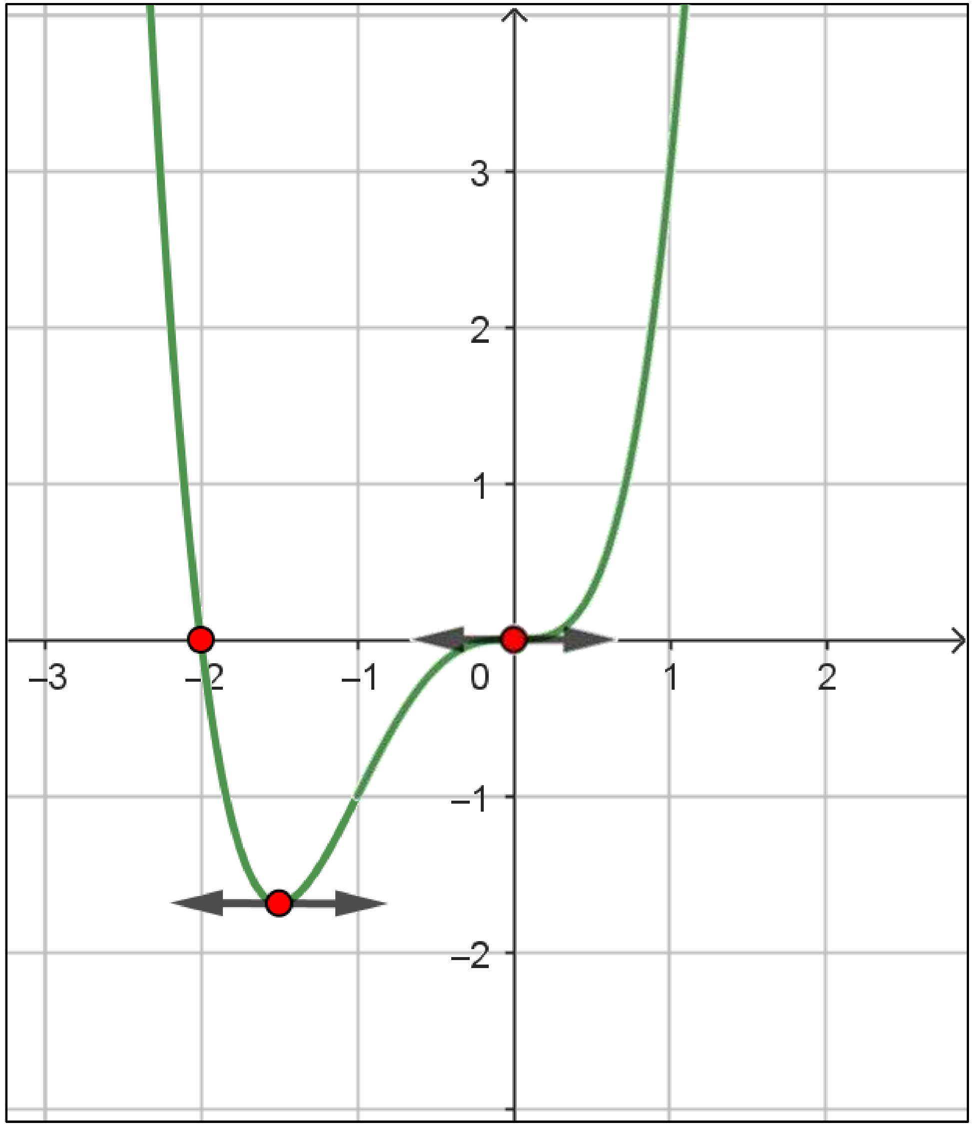


To be more precise, you can calculate $f(1)$ then, you continue drawing.

Consider the function f defined over D=IR by $f(x) = x^4 + 2x^3$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

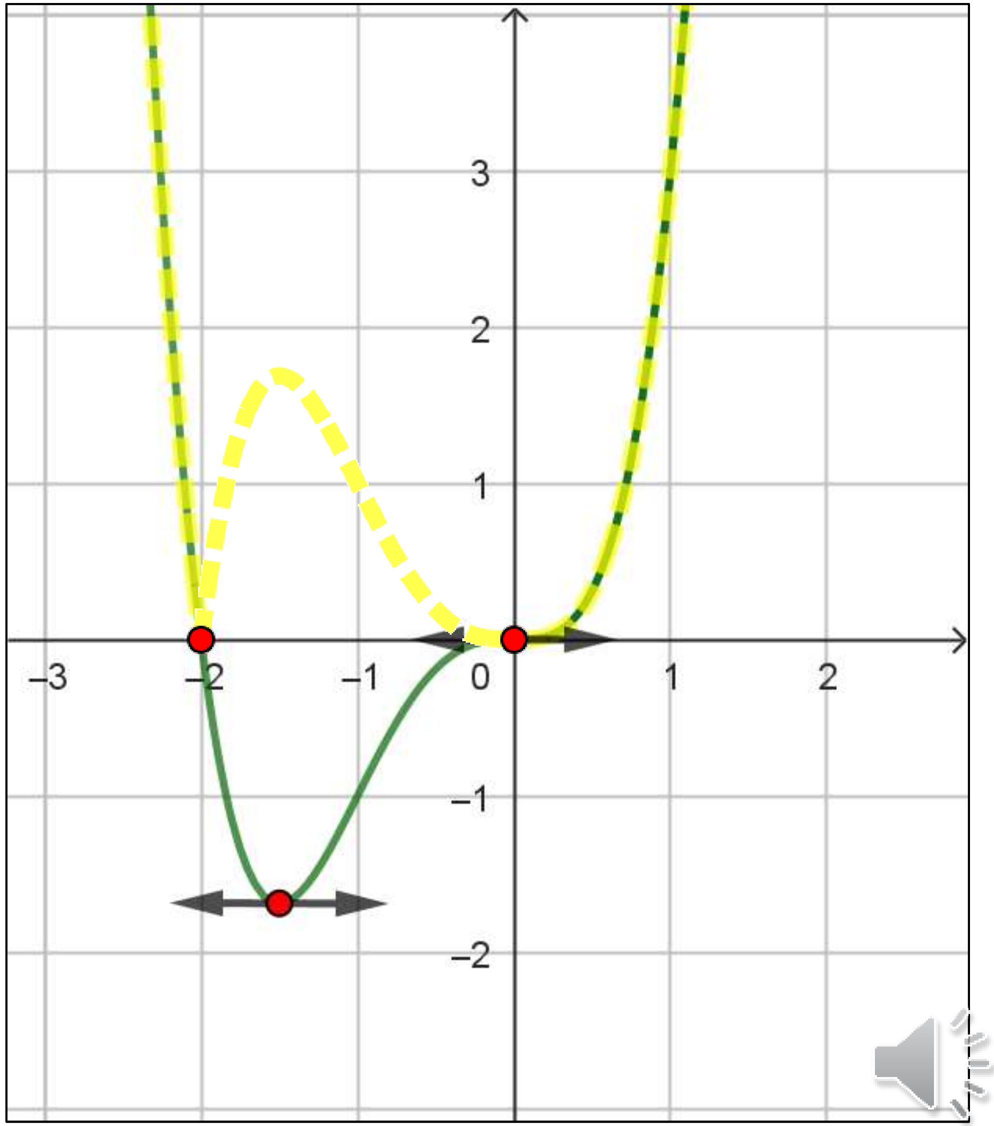
6. Consider the following functions:

a) $h(x) = |f(x)|$

b) $k(x) = f(|x|)$

c) $g(x) = f(x+2)$

How can you deduce the curves of these three functions from (C).



a) $h(x) = \begin{cases} f(x) \text{ if } f(x) \geq 0 \\ x \in]-\infty; -2] \cup [0; +\infty[\\ (C_h) = (C_f) \\ -f(x) \text{ if } f(x) \leq 0 \\ x \in [-2; 0] \\ (C_h) = Sym_{(x'x)}^{(C_f)} \end{cases}$

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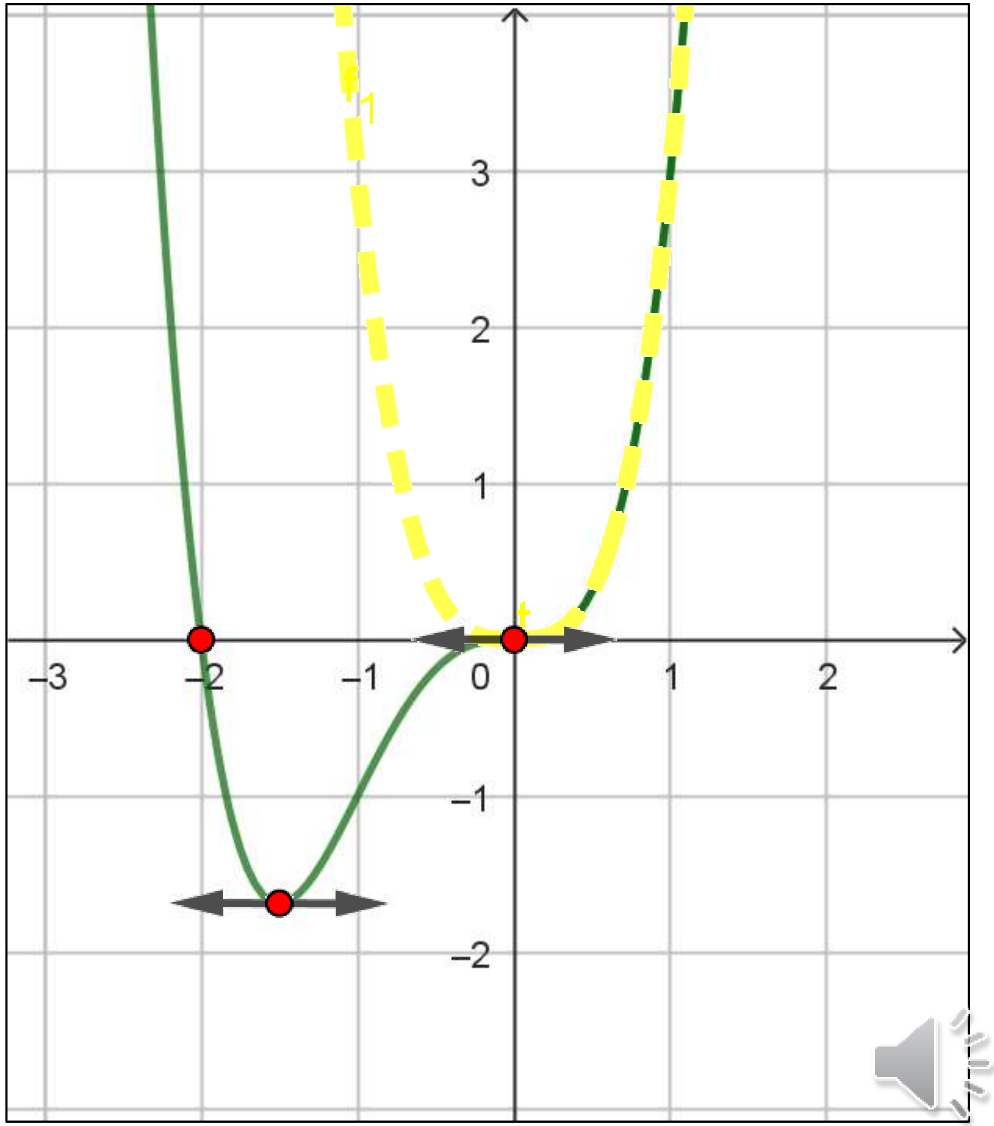
$$\text{b) } k(x) = \begin{cases} f(x) \text{ if } x \geq 0 \\ (C_k) = (C_f) \\ f(-x) \text{ if } x \leq 0 \\ (C_k) = Sym_{(y'y)}^{(C_f)} \end{cases}$$



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c) $g(x) = f(x+2)$

So (C_g) is the translation of (C_f) by the vector (-2;0).